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Quantum Correlations Between a Pair of Photons in a Δ-Type Atom

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Abstract The quantum correlation between a pair of photons, emitted by a Δ-type atom, is calculated. The correlation show anti-bunching property, and the second correlation function is dependent on classical field Ω. So we can control the magnitude of the joint detection probability by adjusting the driving field. In addition, Δ-type atom can also work as storage and retrieving apparatus.

1 Introduction

The generation of high intensity correlation photon source as a current interesting subject has been studied by many authors. Correlated pairs are very useful because it can be used in quantum information such as quantum memory [1], quantum communication [2], and in controllable few-photon pulse generation [3]. Such pairs can also be used for enhancing of resolution in quantum microscopy and lithography [4–6]. The stokes-anti-stokes Raman emission (RED) as a scheme to produce high intensity correlation were discussed in the early works of Agarwal and Jha [7] and Scully and Druhl [8]. A serious of experiments has also been done by several groups [1–3, 9, 10]. The quantum correlation in a three or four level atomic system have shown strong anti-bunching feature and have a controllable correlation [2, 10, 11]. Scully [12] have shown that using strong correlated photon in Raman emission the magnitude of the joint detection probability can be controlled by the classical field.

In this paper, we study two-photon correlation in a three-level atom driven by a classical field. Our three-level atomic system can be a cascade or so-called atomic cyclic Δ-type system [13, 14]. Reference [14] shows us that the three-level atoms with Δ-type transition can be used to perform quantum information transformation because information can be transfered from one quantized optical mode to another. We investigate the two-photon correlation function, and found that the correlation function have the similar behavior to that of RED [12], which means the correlation can be controlled by the driving field Ω. If we

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look q photon as preparing and k photon as retrieving of atomic state, atomic memory can be achieved.

2 Calculation of Two Photon Correlation

In this section, we present the calculation of two photon correlation. We consider a three-level atom driven by a classical field. The atomic level configuration is depicted in Fig. 1. The classical driving field is resonant with the atomic transition $|a\rangle \leftrightarrow |c\rangle$. In the interaction picture, the full Hamiltonian for the scheme shown in Fig. 1 is given by

$$\hat{H} = \hbar\Omega|a\rangle\langle c| + \sum_k \hbar g_k^*(r_0)a_k|a\rangle\langle b|e^{-i(\omega_{ab}-\omega_k)t} + \sum_q \hbar g_q^*(r_0)a_q|b\rangle\langle c|e^{-i(\omega_{bc}-\omega_q)t} + \text{h.c.}, \tag{1}$$

where Ω is Rabi frequency associated with the classical field. $g_k(r_0)$, $g_q(r_0)$ are the coupling frequency for photon emissions between level $|a\rangle \leftrightarrow |b\rangle$ and $|b\rangle \leftrightarrow |c\rangle$. $|i\rangle\langle j|$ $(i, j = a, b, c)$ are the atomic operator, $a_k(a_k^+)$ and $a_q(a_q^+)$ are the annihilation (creation) operators of photon k and photon q.

The wave function method had been used in treating RED [12] as well as CED (cascade emission doublet) [15]. If the reabsorption is weak,the approximation of cutting wave function is used [12]. Here, we also employ this method to calculate the correlation function and cut the wave function at the first cycle. We assume that initial state of the atom is in the state $|b\rangle$ and the field modes are in the vacuum state $|0\rangle$. The corresponding atom-field state of the system can be written as

$$|\Psi(t)\rangle = c_0(t)|b,0\rangle + \sum_q c_{qc}(t)|c,1_q\rangle + \sum_q c_{qa}(t)|a,1_q\rangle + \sum_{kq} c_{kq}(t)|c,1_k,1_q\rangle. \tag{2}$$

Using the Schrödinger equation, we obtain the equations of motion for the probability amplitudes $c_0(t)$, $c_{qa}(t)$, $c_{qc}(t)$ and $c_{kq}(t)$ as

$$i\dot{c_0}(t) = \sum_q g_q^*(r_0)c_{qc}(t)e^{i(\omega_{bc}-\omega_q)t}, \tag{3}$$

$$i\dot{c_{qa}}(t) = \Omega c_{qc}(t) + \sum_k g_k^*(r_0)c_{kq}(t)e^{i(\omega_{ab}-\omega_k)t}, \tag{4}$$

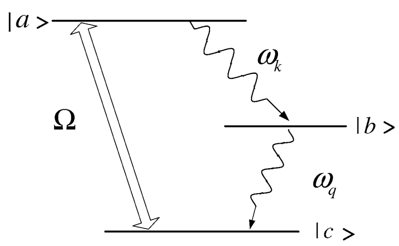

Fig. 1 The Δ-type atomic configuration

$$i\dot{c_{qc}}(t) = \Omega^* c_{qa}(t) + g_q(r_0)c_0(t)e^{-i(\omega_{bc}-\omega_q)t}, \tag{5}$$

$$i\dot{c_{kq}}(t) = g_k(r_0)c_{qa}(t)e^{-i(\omega_{ab}-\omega_k)t}. \tag{6}$$

In order to determine the state of atom and the state of the radiation field, we solve the above equations with the Weisskopf–Wigner approximation [15], and have the relations $\sum_k |g_k(r_0)|^2 = \Gamma_a$, $\sum_q |g_q(r_0)|^2 = \Gamma_b$, where Γ_a and Γ_b means the atomic decay rate from state $|a\rangle$ to $|b\rangle$ and $|b\rangle$ to $|c\rangle$ respectively. We have

$$\dot{c_0}(t) = -\Gamma_b c_0(t), \tag{7}$$

$$\dot{c_{qa}}(t) = -i\Omega c_{qc}(t) - \Gamma_a c_{qa}(t), \tag{8}$$

$$\dot{c_{qc}}(t) = -i\Omega^* c_{qa}(t) - i g_q(r_0)c_0(t)e^{-i(\omega_{bc}-\omega_q)t}, \tag{9}$$

$$\dot{c_{kq}}(t) = -i g_k(r_0)c_{qa}(t)e^{-i(\omega_{ab}-\omega_k)t}, \tag{10}$$

here, $\Omega_d = \sqrt{|\Omega|^2 - (\frac{\Gamma_a}{2})^2}$. Solving the last differential equation, we obtain $c_{qa}(t)$ as

$$\begin{aligned} c_{qa}(t) = \frac{g_q(r_0)\Omega}{2i\Omega_d}\Bigg[&\frac{e^{(-\frac{\Gamma_a}{2}+i\Omega_d)t}}{(\frac{\Gamma_a}{2} - \Gamma_b - i\Delta_2 - i\Omega_d)} - \frac{e^{(-\frac{\Gamma_a}{2}-i\Omega_d)t}}{(\frac{\Gamma_a}{2} - \Gamma_b - i\Delta_2 + i\Omega_d)} \\ &- \frac{2i\Omega_d e^{-(\Gamma_b+i\Delta_2)t}}{(\frac{\Gamma_a}{2} - \Gamma_b - i\Delta_2 - i\Omega_d)(\frac{\Gamma_a}{2} - \Gamma_b - i\Delta_2 + i\Omega_d)}\Bigg], \end{aligned} \tag{11}$$

where $\Delta_1 = \omega_{ab} - \omega_k$, $\Delta_2 = \omega_{bc} - \omega_q$. We are most interested in the state of the field for time $t \gg \Gamma_a^{-1}, \Gamma_b^{-1}$. So we want to know $c_{kq}(\infty)$ as $c_0(\infty)$, $c_{qc}(\infty)$ and $c_{qa}(\infty)$ tend to zero and deduce

$$\begin{aligned} c_{kq}(\infty) = \frac{g_k(r_0)g_q(r_0)\Omega}{2\Omega_d}&\frac{1}{-\Gamma_b - i\Delta_1 - i\Delta_2} \\ &\times\left\{\frac{1}{-\frac{\Gamma_a}{2} + i\Omega_d - i\Delta_1} - \frac{1}{-\frac{\Gamma_a}{2} - i\Omega_d - i\Delta_1}\right\}. \end{aligned} \tag{12}$$

As in the long time limit, $c_0(t)$, $c_{qc}(t)$ and $c_{qa}(t)$ are all zero, and the radiation field is given by

$$|\Psi(\infty)\rangle = \sum_{kq} c_{kq}(\infty)|1_k, 1_q\rangle. \tag{13}$$

The second order two photon correlation function in the atomic cascade emission system is as

$$\begin{aligned} G^{(2)}(1,2) &= \langle\Psi|E_1^{(-)}E_2^{(-)}E_2^{(+)}E_1^{(+)}|\Psi\rangle \\ &= \langle\Psi|E_1^{(-)}E_2^{(-)}|0\rangle\langle 0|E_2^{(+)}E_1^{(+)}|\Psi\rangle \\ &= |\Psi^{(2)}(1,2)|^2, \end{aligned} \tag{14}$$

where $\Psi^{(2)}(1,2) = \langle 0|E_2^{(+)}E_1^{(+)}|\Psi\rangle$ in which $E^{(+)}$ is the positive frequency (annihilation operator) part of the electric field operator.

We first detect q photon then k photon, finally we have

$$\Psi^{(2)}(r_1,t_1;r_2,t_2) = \eta e^{-(i\omega_{bc}+i\omega_{ab}+\Gamma_b)\tau_1}\Theta(\tau_1)e^{-(\frac{\Gamma_a}{2}+i\omega_{ab})(\tau_2-\tau_1)}\sin[\Omega_d(\tau_2-\tau_1)] \times \Theta(\tau_2-\tau_1) + (1 \leftrightarrow 2), \tag{15}$$

where $\eta = \frac{2i\kappa\Omega}{\Delta r_1 \Delta r_2 \Omega_d}$, $\kappa = (2\pi)^4 k_0 g_{a,k_0} g_{b,q_0} \sigma(k_0)\sigma(q_0)$, so the corresponding second order correlation is

$$G^{(2)}(r_1,t_1;r_2,t_2) = |\eta|^2 e^{-\Gamma_b\tau_1}\Theta(\tau_1)e^{-\Gamma_a(\tau_2-\tau_1)} \times \sin^2[\Omega_d(\tau_2-\tau_1)]\Theta(\tau_2-\tau_1) + (1 \leftrightarrow 2). \tag{16}$$

This is the main result of our calculation and we will discuss the behavior of the correlation function.

3 Result and Discussion

We plot the second correlation function in Fig. 2 with different field strengths. From the figure, we can clearly see that curves of correlation function vanish at $\tau_2 - \tau_1 = 0$, indicating the anti-bunching. This is because once the q photon is emitted, the atom goes to the state $|c\rangle$ and needs to be pumped to the state $|a\rangle$ for emission of k photon. Thus, for $\Omega > \frac{\Gamma_a}{4}$, the two-photon correlation is also a oscillation function of the pumping Ω_d. A decaying oscillation is a quantum interference effect or a Rabi oscillation of the atom between $|b\rangle$ and $|a\rangle$. One can also see that quantum interference is destructed when $\tau_2 - \tau_1 = \frac{n\pi}{\Omega_d}$ ($n = 0, 1, 2, \ldots$), and correlation becomes zero. If $\Omega = \frac{\Gamma_a}{4}$, the oscillation factor $\sin[\Omega_d(\tau_2 - \tau_1)]/\Omega_d \to \tau_2 - \tau_1$, from (14) and (15), one can easily see $G^{(2)}(r_1,t_1;r_2,t_2) \to \Omega(\tau_2 - \tau_1)^2 e^{-\frac{\Gamma_b}{2}\tau_1}\Theta(\tau_1)e^{-\frac{\Gamma_a}{2}(\tau_2-\tau_1)}\Theta(\tau_2-\tau_1) + (1 \leftrightarrow 2)$ (see Fig. 2(b), here we plot it under the case $\Omega = \frac{\Gamma_a}{2}$).

In the case of $\Omega < \frac{\Gamma_a}{2}$, using the similar manner $G^{(2)}(r_1,t_1;r_2,t_2)$ becomes

$$G^{(2)}(r_1,t_1;r_2,t_2) \to e^{-\Gamma_a(\tau_2-\tau_1)}\sinh^2[\Omega_d^{'}(\tau_2-\tau_1)]\Theta(\tau_2-\tau_1) + (1 \leftrightarrow 2), \tag{17}$$

with $\Omega_d^{'} = \sqrt{(\Gamma_a/2)^2 - |\Omega|^2}$. In this case, namely over-damped case $G^{(2)}(r_1,t_1;r_2,t_2)$ is very weak shown in Fig. 2(c). It is understandable in physics because if classical Ω is

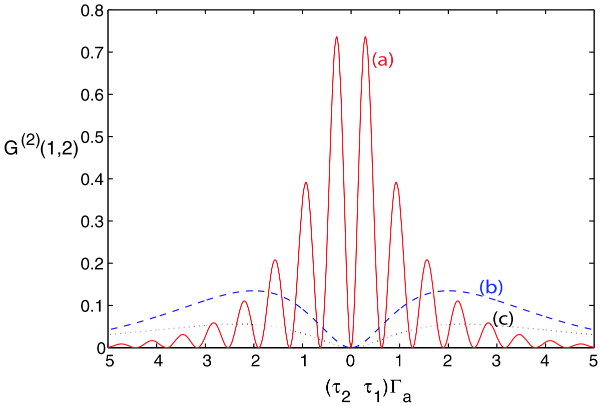

Fig. 2 (Color online) The two photons correlation function $G^{(2)}(r_1,t_1;r_2,t_2)$ for a cyclic Δ-type atom as a function of retarded time $\tau_2 - \tau_1$. The three curves is corresponding to three differently cases (a) $\Omega = 5\Gamma_a$; (b) $\Omega = 0.5\Gamma_a$; (c) $\Omega = 0.4\Gamma_a$

too weak, correlation will also be weak. Therefore, one can conclude that the correlation are strongly affected by the classical strength Ω. On the other hand, although our atomic configuration can also be viewed as a cascade system, our system shows anti-bunching, different with usually bunching behavior of CED [15]. This is because the different initial condition and the introduction of classical field. The most important difference is that our system can be used as quantum memory while usually CED can not. We will state it in succeeding paragraph.

Now we want to discuss potential application of our Δ-type atomic system in quantum information. As it was reported in Ref. [1], a atomic ensemble with "lambda" configuration, coupled by a pair of optical controls fields, can work as information storage and retrieval because of Stokes photon and Anti-Stokes photon correlation. In our scheme, if we code information in state $|b\rangle$ when we prepare the initial state, the information will be carried in q photon when the atom goes to state $|c\rangle$; and the information is stored in state $|c\rangle$. In the second process, after the retrieve field Ω drive the atom from $|c\rangle$ to $|a\rangle$, k photon field will be created and the information can be retrieved. The storage time (delay time) and the rate of retrieval are controlled by the intensity of field Ω. So, Δ-type atomic system also can also promise to work as quantum information apparatus.

4 Conclusion

We have calculated the two-photon correlation between a pair of photons in a Δ-type atomic system. According to the result of calculation, anti-bunching behavior of photons is shown in the cyclic three-level atom. Another feature is that the second correlation function $G^2(r_1, t_1; r_2, t_2)$ is dependent on classical field Ω. So we can control the magnitude of the joint detection probability by the driving field. In addition, if we look on q photon as preparing and k photon as retrieving of atomic state, quantum information process can be achieved.

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